CS 188: Artificial Intelligence
Spring 2010

Lecture 8: MEU / Utilities
2/11/2010

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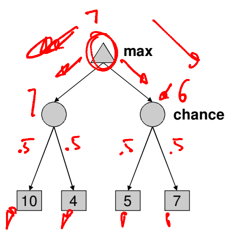
Many slides over the course adapted from Dan Klein

Announcements

- W2 is due today (lecture or drop box)
- P2 is out and due on 2/18

Expectimax Search Trees

- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, next card is unknown
  - In minesweeper, mine locations
  - In pacman, the ghosts act randomly
- Can do **expectimax search**
  - Chance nodes, like min nodes, except the outcome is uncertain
  - Calculate expected utilities
  - Max nodes as in minimax search
  - Chance nodes take average (expectation) of value of children
- Later, we'll learn how to formalize the underlying problem as a **Markov Decision Process**



Maximum Expected Utility

- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility: an agent should choose the action which maximizes its expected utility, given its knowledge
- General principle for decision making
- Often taken as the definition of rationality
- We'll see this idea over and over in this course!
- Let's decompress this definition…
  - Probability --- Expectation --- Utility

Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes
- Example: traffic on freeway?
  - Random variable: T = amount of traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.55, P(T=heavy) = 0.20
- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one
- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.20, $\boxed{\text{P(T=heavy | Hour=8am)}}$ = 0.60
  - We'll talk about methods for reasoning and updating probabilities later

What are Probabilities?

- Objectivist / frequentist answer:
  - Averages over repeated *experiments*
  - E.g. empirically estimating P(rain) from historical observation
  - Assertion about how future experiments will go (in the limit)
  - New evidence changes the *reference class*
  - Makes one think of *inherently random* events, like rolling dice
- Subjectivist / Bayesian answer:
  - Degrees of belief about unobserved variables
  - E.g. an agent's belief that it's raining, given the temperature
  - E.g. pacman's belief that the ghost will turn left, given the state
  - Often *learn* probabilities from past experiences (more later)
  - New evidence *updates beliefs* (more later)

Uncertainty Everywhere

- Not just for games of chance!
  - I'm sick: will I sneeze this minute?
  - Email contains "FREE!": is it spam?
  - Tooth hurts: have cavity?
  - 60 min enough to get to the airport?
  - Robot rotated wheel three times, how far did it advance?
  - Safe to cross street? (Look both ways!)
- Sources of uncertainty in random variables:
  - Inherently random process (dice, etc)
  - Insufficient or weak evidence
  - Ignorance of underlying processes
  - Unmodeled variables
  - The world's just noisy – it doesn't behave according to plan!

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Reminder: Expectations

- We can define function f(X) of a random variable X
- The expected value of a function is its average value, weighted by the probability distribution over inputs
- Example: How long to get to the airport?
  - Length of driving time as a function of traffic: L(T)
    L(none) = 20, L(light) = 30, L(heavy) = 60
  - What is my expected driving time?
    - Notation: E[L(T)]
    - Remember, P(T) = {none: 0.25, light: 0.5, heavy: 0.25}
    - E[L(T)] = L(none) * P(none) + L(light) * P(light) + L(heavy) * P(heavy)
    - E[L(T)] = (20 * 0.25) + (30 * 0.5) + (60 * 0.25) = 35

      = 20 * .25 + 30 * .5 + 60 * .25
      = 35

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Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any set of preferences between outcomes can be summarized as a utility function (provided the preferences meet certain conditions)
- In general, we hard-wire utilities and let actions emerge (why don't we let agents decide their own utilities?)
- More on utilities soon…

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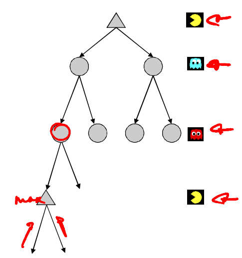
Expectimax Search

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a node for every outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!
- For now, assume for any state we magically have a distribution to assign probabilities to opponent actions / environment outcomes



Having a probabilistic belief about an agent's action does not mean that agent is flipping any coins!

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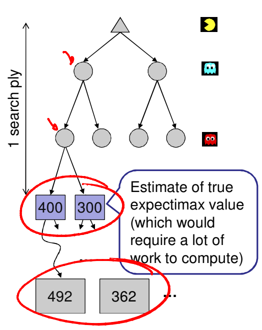
Expectimax Search

- Chance nodes
  - Chance nodes are like min nodes, except the outcome is uncertain
  - Calculate expected utilities
  - Chance nodes average successor values (weighted)
- Each chance node has a probability distribution over its outcomes (called a model)
  - For now, assume we're given the model
- Utilities for terminal states
  - Static evaluation functions give us limited-depth search

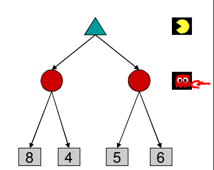


Expectimax Pseudocode

```
def value(s)
    if s is a max node return maxValue(s)
    if s is an exp node return expValue(s)
    if s is a terminal node return evaluation(s)

def maxValue(s)
    values = [value(s') for s' in successors(s)]
    return max(values)

def expValue(s)
    values = [value(s') for s' in successors(s)]
    weights = [probability(s, s') for s' in successors(s)]
    return expectation(values, weights)
```



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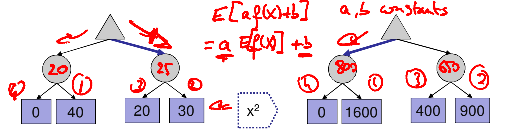
Expectimax Evaluation

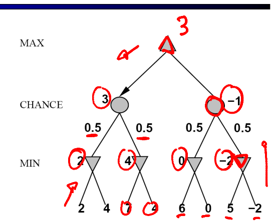
- Evaluation functions quickly return an estimate for a node's true value (which value, expectimax or minimax?)
- For minimax, evaluation function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For expectimax, we need *magnitudes* to be meaningful

E[af(x)+b] = a E[f(x)] + b   a, b constants



Mixed Layer Types

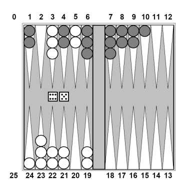
- E.g. Backgammon
- Expectiminimax
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax



ExpectiMinimax-Value(*state*):
- **if** *state* is a MAX node **then**
  - **return** the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
- **if** *state* is a MIN node **then**
  - **return** the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
- **if** *state* is a chance node **then**
  - **return** average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

Stochastic Two-Player

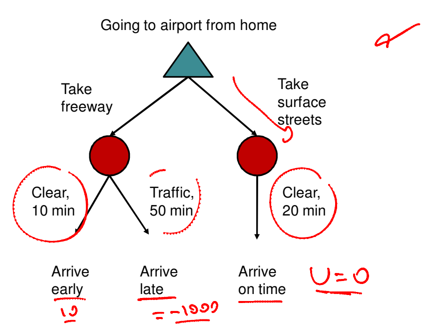
- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 4 = 20 x (21 x 20)^3 1.2 x 10^9
- As depth increases, probability of reaching a given node shrinks
  - So value of lookahead is diminished
  - So limiting depth is less damaging
  - But pruning is less possible…
- TDGammon uses depth-2 search + very good eval function + reinforcement learning: world-champion level play



Maximum Expected Utility

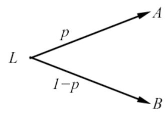
- Principle of maximum expected utility:
  - A rational agent should choose the action which maximizes its expected utility, given its knowledge
- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - Why are we taking expectations of utilities (not, e.g. minimax)?
  - What if our behavior can't be described by utilities?

Utilities: Unknown Outcomes



Preferences

- An agent chooses among:
  - Prizes: A, B, etc.
  - Lotteries: situations with uncertain prizes



$$L = [p, A;\ (1-p), B]$$

- Notation:

$A \succ B$	A preferred over B
$A \sim B$	indifference between A and B
$A \succeq B$	B not preferred over A

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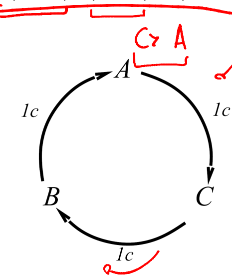
Rational Preferences

- We want some constraints on preferences before we call them rational

AND

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

- For example: an agent with intransitive preferences can be induced to give away all of its money
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C



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Rational Preferences

- Preferences of a rational agent must obey constraints.
  - The axioms of rationality:

Orderability
$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$
Transitivity
$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$
Continuity
$$A \succ B \succ C \Rightarrow \exists p\ [p, A;\ 1-p, C] \sim B$$
Substitutability
$$A \sim B \Rightarrow [p, A;\ 1-p, C] \sim [p, B; 1-p, C]$$
Monotonicity
$$A \succ B \Rightarrow$$
$$(p \geq q \Leftrightarrow [p, A;\ 1-p, B] \succeq [q, A;\ 1-q, B])$$

- Theorem: Rational preferences imply behavior describable as maximization of expected utility

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MEU Principle

- Theorem:
  - [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \Leftrightarrow A \succeq B$$
$$U([p_1, S_1;\ \ldots\ ;\ p_n, S_n]) = \sum_i p_i U(S_i)$$

- Maximum expected utility (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tictactoe

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Utility Scales

- Normalized utilities: $u_+ = 1.0$, $u_- = 0.0$
- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.
- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

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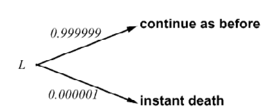
Human Utilities

- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment of human utilities:
  - Compare a state A to a standard lottery L_p between
    - "best possible prize" u_+ with probability p
    - "worst possible catastrophe" u_- with probability 1-p
  - Adjust lottery probability p until A ~ L_p
  - Resulting p is a utility in [0,1]

[p, u+, (1-p), u-]

pay $30 ~



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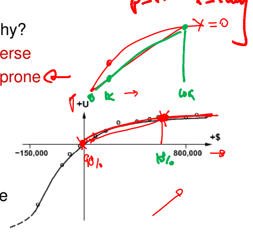
Money

$U([p,A),(1-p,B]) = pU(A)+(1-p)U(B)$

- Money does not behave as a utility function, but we can talk about the utility of having money (or being in debt)
- Given a lottery L = [p, $X; (1-p), $Y]
  - The expected monetary value EMV(L) is p*X + (1-p)*Y = 1000
  - U(L) = p*U($X) + (1-p)*U($Y)
  - Typically, U(L) < U(EMV(L)): why?
  - In this sense, people are risk-averse
  - When deep in debt, we are risk-prone
- Utility curve: for what probability p am I indifferent between:
  - Some sure outcome x
  - A lottery [p,$M; (1-p),$0], M large



Example: Insurance

- Consider the lottery [0.5,$1000; 0.5,$0]
  - What is its expected monetary value? ($500)
  - What is its certainty equivalent?
    - Monetary value acceptable in lieu of lottery
    - $400 for most people
  - Difference of $100 is the insurance premium
    - There's an insurance industry because people will pay to reduce their risk
    - If everyone were risk-neutral, no insurance needed!

Example: Insurance

- Because people ascribe different utilities to different amounts of money, insurance agreements can increase both parties' expected utility

You own a car. Your lottery:
L_Y = [0.8, $0 ; 0.2, -$200]
i.e., 20% chance of crashing

You do not want -$200!

$U_Y(L_Y) = 0.2*U_Y(-\$200) = -200$
$U_Y(-\$50) = -150$

Amount	Your Utility U_Y
$0	0
-$50	-150
-$200	-1000

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Insurance company buys risk:
L_I = [0.8, $50 ; 0.2, -$150]
i.e., $50 revenue + your L_Y

Insurer is risk-neutral:
U(L)=U(EMV(L))

$U_I(L_I) = U(0.8*50 + 0.2*(-150))$
$= U(\$10) > U(\$0)$

Example: Human Rationality?

- Famous example of Allais (1953)
  - A: [0.8,$4k; 0.2,$0]
  - B: [1.0,$3k; 0.0,$0]
  - C: [0.2,$4k; 0.8,$0]
  - D: [0.25,$3k; 0.75,$0]
- Most people prefer B > A, C > D
- But if U($0) = 0, then
  - B > A ⇒ U($3k) > 0.8 U($4k)
  - C > D ⇒ 0.8 U($4k) > U($3k)